Publication Date: 30 June 2024
Archs Sci. (2024) Volume 74, Issue 3 Pages 114-117, Paper ID 2024319.
https://doi.org/10.62227/as/74319

Core-Polarization Effects on Electron Scattering with Modified Surface Delta Interaction Using Nuclear Shell Model Calculation for ^{10}B, ^{19}F and ^{39}K Nuclei

Khilood H. Ali[1,*] and Khalid S. Jassim[1]
[1]Department of Physics, College of Education for Pure Sciences, Babylon University, Babylon, Iraq.
Corresponding authors: Khilood H. Ali (e-mail: uobabylon.edu.iq).

Abstract The study of NN interactions involves the calculation through inelastic electron scattering from longitudinal shape factors. The factors that come into play in the determination of the longitudinal shape factor for inelastic electron scattering are the transfer momentum and angular momentum. We delve into the analysis of electron scattering shape factors as well as CP — standing for nuclear polarization — in longitudinal inelastic shell models. This is done using NuShellX, where we employ a sum potential HO to look at ^{10}B, ^{39}K, and ^{19}F nuclei. Among the computations carried out include those of Single Particle Matrix Elements and Inelastic Electron Scattering Shape Factors which are then pitted against experimental data. The outcomes depict a scenario where the model space provides a full explanation on NushellX shape factor compared to nuclear polarization; this thereby implies less contribution from certain quarters towards shaping this factor. On another front, the comparison shows that results from longitudinal form factor polarization for NuShellX core find resonance with what is already available experimentally— a pointer towards reliability if not outright correctness.

Index Terms electron Scattering, MSDI, form factor, P-shell, SD-shell, NushellX program

I. Introduction

Atomic scattering is a technique used to gather information on nuclear structure, including the spatial distribution of charge, size and electromagnetic current enclosed within nuclei. The study took off in 1929 [1] when Mott discovered relativistic Dirac particle scattering cross section, and it marked the beginning of theoretical investigations into electron scattering. The size of a nucleus can be deduced by obtaining the product of Mott's cross section with what is known as a "form factor"; this factor describes how current charge and magnetization are distributed within a nucleus [2]. Attempts at accommodating the actual size involve scaling up Mott's cross section by another factor also dependent on these same distributions [3] named as "nuclear form factor." We can empirically determine this form factor through specific observables like scattering angle, incoming energies or momentum transferred to target nucleus. Different models have been proposed to describe nuclei based on their characteristic features (such as static properties); one widely used model space approach (for explaining high momentum transfer data) fails despite its success in certain aspects towards end regions where more configurations need be included for improvement [4]. Numerous shell model codes have addressed the eigenvalue problem in shell model computations, for exampleThe USD Hamiltonian, known as "OXBASH," "ANTOINE," "NUSHELL," and "NUSHELLX," has been utilized for over twenty years in nuclear astrophysics, nuclear spectroscopy, and models of nuclear structure, providing realistic wave functions for the sd-shell (1d5/2, 1d3/2, 2s1/2). It is a crucial component in both p-sd and sd-pf spaces. In reference [5], resulting in a 63-entry two-body matrix. The USD-type interaction, involving USDA interactions, is determined by fitting sixty-six parameters to energy data within A=16-40 nuclei, including all oxygen isotopes and resolving the fluorine issue. The rms divergence for the respective isotopes is 130 keV and 170 keV. Various theoretical articles on electron scattering shape factors in psd sdpf or psd shells have been studied including 10B, 39K and 19F as shown in Ref. [6], [7]. The NushellX CP algorithms were used to compare the computed coulomb plus longitudinal form factors against experimental data.

A. Theory of electron scattering

The interpretation of the "electron scattering form factor" is based on a specific definition of multilateralism and momen-

ARCHIVES DES SCIENCES
A Multidisciplinary Journal

tum transfer [8],

$$|F_\lambda(q)|^2 = \frac{1}{2j_i+1}\left(\frac{4\pi}{Z^2}\right) \times \left|\left\langle \Gamma_f \, ||| \, \hat{T}_\lambda^\xi \, ||| \, \Gamma_i \right\rangle\right|^2 | \, F_{f.s}F_{c.m} \, |^2, \quad (1)$$

where $F_{f.s} = e^{-0.43q^2/4}$ the fixed modification of nucleon size and $F_{c.m} = q^2b^2/4A$ The The resonator's scaling factor, b, is used to denote the mass number and is influenced by the mass correction. The form factor is affected by nuclear polarization (CP), which is derived from a microscopic theory that combines shell model functions with waves and higher energy configurations. This theory expands upon the intrusion of particle-hole interactions."The matrix component of the electron scattering operator The notation below indicates the contribution coming from "fp-model space" to the input quantity and joining "core-polarization (CP)" [8];

$$\left\langle \Gamma_f \, ||| \, \hat{T}_\lambda^\xi \, ||| \, \Gamma_i \right\rangle = \left\langle \Gamma_f \, ||| \, \hat{T}_\lambda^\xi \, ||| \, \Gamma_i \right\rangle_{ms} + \left\langle \Gamma_f \, ||| \, \delta\hat{T}_\lambda^\xi \, ||| \, \Gamma_i \right\rangle_{cp}. \quad (2)$$

Choose ξ, the transverse form factors, denoted as L, E, and M, were found to correspond to the longitudinal, electric, and magnetic components. The Greek symbols were observed to signify quantum numbers in both coordinate space and isospace. It is permissible to consider the linear combination of single-particle matrix elements as the element within the fp-shell model-space (MS) [9].

$$\left\langle \Gamma_f \, ||| \, \hat{T}_\lambda^\xi \, ||| \, \Gamma_i \right\rangle_{ms} = \sum_{\alpha_f\alpha_i} \chi^\lambda \Gamma_f \Gamma_i(\alpha_f,\alpha_i) \left\langle \, \alpha_f \, ||| \, \hat{T}_\lambda^\xi \, ||| \, \alpha_i \, \right\rangle, \quad (3)$$

where $\chi^\lambda \Gamma_f \Gamma_i(\alpha_f,\alpha_i)$ structure factors (a single body density matrix element) are supplied by:

$$\chi^\lambda \Gamma_f \Gamma_i(\alpha_f,\alpha_i) = \frac{\left\langle \Gamma_f \, ||| \, [a^+(\alpha_f) \otimes \tilde{a}(\alpha_i)]^\lambda \, ||| \, \Gamma_i \, \right\rangle}{\sqrt{2\lambda+1}}. \quad (4)$$

In the p-shell model space (MS), theα_i and α_f label are both single particle states. Consequently, the element's matrix of core polarization (CP) is as follows:

$$\left\langle \Gamma_f \, ||| \, \delta\hat{T}_\lambda^\xi \, ||| \, \Gamma_i \right\rangle_{cp} = \sum_{\alpha_f\alpha_i} \chi^\lambda \Gamma_f \Gamma_i(\alpha_f,\alpha_i) \left\langle \, \alpha_f \, ||| \, \delta T_\lambda^\xi \, ||| \, \alpha_i \, \right\rangle. \quad (5)$$

For higher-energy configuration, the "single-particle matrix aspect" is provided by [10] up to the "first level expansion method".

$$\langle \, \alpha_f \, ||| \, \delta T_\lambda^\xi \, ||| \, \alpha_i \, \rangle = \langle \, \alpha_f \, ||| \, \hat{T}_\lambda^\xi \frac{Q}{E_i - H_o} V_{res} \, ||| \, \alpha_i \, \rangle + \langle \, \alpha\Gamma_f \, ||| \, V_{res} \frac{Q}{E_f - H_o} \hat{T}_\lambda^\xi \, ||| \, \alpha\Gamma_i \, \rangle. \quad (6)$$

Operator Q is the space projection operator outside the model's space. The energies are of E_i and E_f in both the initial and final states. The two terms for the remaining relationship V_{res}, MSDI and M3Y, can be written as [12] on the right wrist's side using Eq. (6):

$$\sum_{\alpha_1\alpha_2\Gamma} \frac{(-1)^{\alpha_i+\alpha_2+\Gamma}}{e_{\alpha_i} - e_{\alpha_f} - e_{\alpha_1} + e_{\alpha_2}} (2\Gamma+1) \times \begin{Bmatrix} \alpha_f & \alpha_i & \lambda \\ \alpha_2 & \alpha_1 & \Gamma \end{Bmatrix} \sqrt{(1+\delta_{\alpha_1\alpha_f})(1+\delta_{\alpha_2\alpha_i})} \times \langle \alpha_2 \, \| \, T_\lambda \, \| \, \alpha_1 \rangle \times \langle \alpha_f \alpha_1 | V_{res} | \alpha_i \alpha_2 \rangle_\Gamma$$
$$+ \text{Terms with } \alpha_1 \text{ and } \alpha_2 \text{ exchanged with an over all minus sign}, \quad (7)$$

where e the energy of a single particle is played with by α_1 particle states andα_2 hole states. The conditions of the hole-states and1s the core orbits shield every state

$$e_{nlj} = (2n + l - \frac{1}{2})\hbar\omega + \begin{cases} -\frac{1}{2}(l+1)\,\langle f(r) \rangle_{nl} & \text{for} \quad j = l - \frac{1}{2}, \\ \frac{1}{2} l \, \langle f(r) \rangle_{nl} & \text{for} \quad j = l + \frac{1}{2}, \end{cases} \quad (8)$$

where $\langle f(r) \rangle_{nl} \approx -20A^{-2/3}$ and $h\omega = 45A^{-1/3} - 25A^{-2/3}$

Strength for the electric of transition is determined from:

$$B(C\lambda) = \frac{|(2\lambda+1)!!\,|^2}{4\pi} \frac{Z^2}{k^\lambda} \left|F_\lambda(k)\,\right|^2, \quad (9)$$

where $k = E_x \, / \hbar c$ is the wave number.

II. Results and Discussion

A. The nucleus ^{10}B

The model space p shells have effective interactions in ckpot and "Core polarization (CP) measurements with equal scalar changes on a state basis" ($J\pi$ = $3_1{}^+$, T = 0) to the state ($J\pi$ = 1+, T = 0) effect. We use EX = 0.0718 MeV of the core ^{10}B-C2 node in core ^{10}B as the excitation energy to determine the possible resonator action through the interaction matrix elements; b (size parameter value of 1.75 fm) is applied to the single particle wave function. See Figure 1 for details. ., One peak is seen in the model space (MS) (without the (cp) effect) that deviates from the experimental data in the momentum transfer range of 0.4 to 1.4 fm-1. It is noteworthy that the model-space (MS) falls between 1.4 and 2.3 fm-1. It is exactly in line with the experimental data ,We can see that the core polarization (cp) is overestimated at momentum transform in the region 1.1 to 1.8 fm-1 and underestimated in the region 0.4 to 0.9 fm-1. The experimental data is in good agreement in the region 1.8 to 2.3 fm-1. Radhi, et al. [11] .The entire model space and core polarization of the experimental data are underestimated in all momentum transfer regions.

B. The nucleus ^{39}K

Use the C2 method to measure the ground state transition ($J\pi$ = $3/2_1{}^+$, T = 1/2) to ($J\pi$ = 1/2). The single particle wave equation (HO) is used with size parameters b = 1.69 fm and T = 1/2. The measurement was performed at EX = 2.523

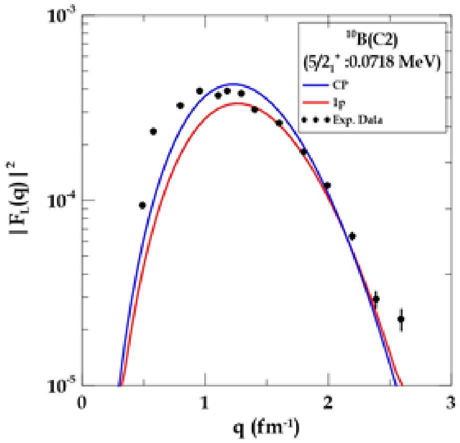

Figure 1: The ^{10}B isotope form factor's longitudinal C2 graph (with and without core-polarization effects.) . For the ground state ($J^\pi = 3_1{}^+$ and T = 0), isoscalar transformation to ($J^\pi = 1^+$ and T = 0), E_X = 0.0718 MeV

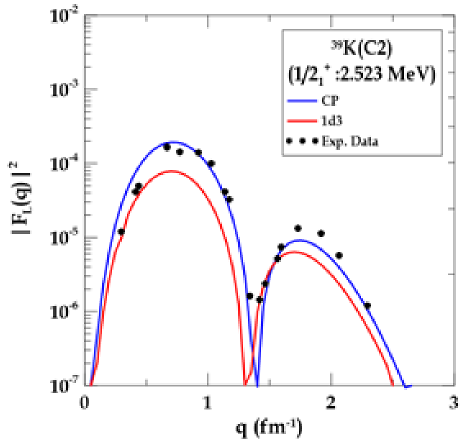

Figure 2: The longitudinal form factor of the ^{39}K isotope during the C2 transition(with and without core-polarization effects.).. At E_X = 2.523 MeV, the isoscalar transformation from the ground of state ($J^\pi = 3/2_1{}^+$,T = 1/2) to the state (J^π = 1/2, T = 1/2) occurs

MeV. By using the elements in the interaction matrix (OBDM) shown in Figure 2 and the SD shell and SDM interactions and core polarization (CP), Without the cp effect, the model space (MS) is highly arranged in the area where the experimental results are between 0.3 and 0.5 fm-1 and 1.3 and 1.6 fm-1, and it deviates from

Figure 2, displays experimental data of momentum transfer ranging from 0.5 to 1.3 fm-1. The experimental data in the 0.3 to 0.7 fm1 region of momentum transfer overestimates the core polarization (cp). As seen in Figure 2 by Radhi et al. [11], the inclusion of cp effects significantly increases the C2 form factor and the core polarization has a good agreement with the experimental data in the momentum transfer region (0.7 – 1.4) fm-1. However, it has no effect on the total form factor because of its small contribution to the total form factor in the region of q < 2 fm-1.

C. The nucleus ^{19}F

"In the course of determining the harmonic potential function for a single-particle wave function, an OBDM usage of interaction matrix elements involves transforming the C2 measurement value of on a state basis ($J\pi$ = 1/2, T = 1/2) into a scalar state at EX = 0.19 MeV ($J\pi = 5/2_1{}^+$) and with size parameter b = 1.88 fm— , The core polarization (CP) is represented by the blue curve. the red curve represent the model space (MS) , We note that the model-space (MS) in the region of 1.8 to 2 fm-1 shows deviation from experimental data between 0.8 and 1.8 fm-1 of momentum transfer (without the cp effect). It perfectly lines up with the experimental data shown in Fig. 3. The core polarization has a good agreement with the experimental data in the region (0.8 – 1.8) fm-1 of the momentum transfer, and we notice from Fig. 3, that it is overestimated in the region 1.8 to 3 fm-1 at momentum transform. The inclusion of cp effects improves the C2 form factor noticeably. Radhi, et al. [11] The entire model space and core polarization of the experimental data are underestimated

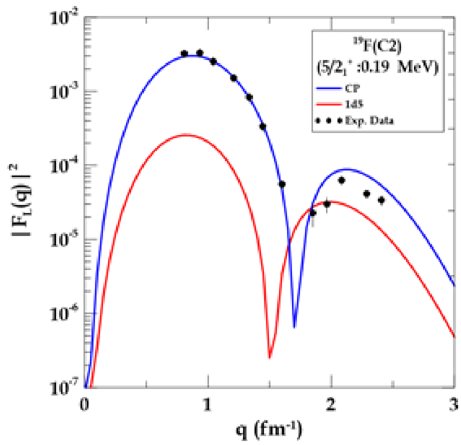

Figure 3: The ^{19}F isotope type factor's longitudinal C2 is shown in Figure 3(with and without core-polarization effects.). At E_X = 0.19 MeV ($J^\pi = 5/2_1{}^+$), the isoscalar transformation to T = 1/2 states via the ground of state (J^π = 1/2 and T = 1/2)

in all momentum transfer regions.

III. Conclusions

The shape factor is the most effective residual interaction if the harmonic oscillator (HO) is used. calculations are done with specified effective proton and neutron charges. In trying to Obtaining two models is not possible when trying to replicate the experimental data in the high momentum transfer region (q > 2 fm-1).p-shell model and SD-shell model because transformation energy level of C2 does not match electron scattering formation factor, thus high configurations influence on microscopic range space of p-shell and SD-shell contains the theory which is called nucleopolar chemical effect (cp).

References

[1] Francis, N. M. (1929). The scattering of fast electrons by atomic nucleiProc. *R. Soc. Lond. A124*, 425–442

[2] Brown, B. A., Radhi, R., & Wildenthal, B. H. (1983). Electric quadrupole and hexadecupole nuclear excitations from the perspectives of electron scattering and modern shell-model theory. *Physics Reports, 101*(5), 313-358.

[3] Sagawa, H., & Brown, B. A. (1984). E2 core polarization for sd-shell single-particle states calculated with a skyrme-type interaction. *Nuclear Physics A, 430*(1), 84-98.

[4] Delorme, J., Figureau, A., & Guichon, P. (1981). Nuclear critical opalescence and the M1 form factors of 12C and 13C. *Physics Letters B, 99*(3), 187-190.

[5] Bertsch, G., Borysowicz, J., McManus, H., & Love, W. G. (1977). Interactions for inelastic scattering derived from realistic potentials. *Nuclear Physics A, 284*(3), 399-419.

[6] Liu, J., Zhang, J., Xu, C., & Ren, Z. (2017). Nuclear longitudinal form factors for axially deformed charge distributions expanded by nonorthogonal basis functions. *Chinese Physics C, 41*(5), 054101.

[7] Liu, J., Xu, C., Wang, S., & Ren, Z. (2017). Coulomb form factors of odd-A nuclei within an axially deformed relativistic mean-field model. *Physical Review C, 96*(3), 034314.

[8] Caurier, E., & Nowacki, F. (1999). Present status of shell model techniques. *Acta Physica Polonica B, 30*(3), 705.

[9] Brown, B. A., & Rae, W. D. (2007). No TitleB. A. Brown and W. D. Rae, MSU-NSCL Report.

[10] Brown, B. A., & Rae, W. D. (2008). "No Title," , Resour.

[11] Radhi, R. A., Abdullah, A. A., Dakhil, Z. A., & Adeeb, N. M. (2001). Core-polarization effects on C2 form factors of p-shell nuclei. *Nuclear Physics A, 696*(3-4), 442-452.

[12] B. H. Wildenthal, "No Title," Prog. Part. Nucl. Phys., vol. 11, no. 5, 1984.
.