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A Temporal–Graph Fusion Framework for Financial Distress Early Warning in Advanced Manufacturing Enterprises

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Abstract Economic uncertainty and increasingly interconnected supply chains create challenges for identifying financial distress among advanced manufacturing enterprises, where risks may evolve over time and extend across firm relationships. Conventional early warning approaches often have difficulty capturing both the temporal development of financial conditions and the structural dependencies through which vulnerabilities can spread. This study proposes a temporal–graph fusion deep learning framework for financial distress early warning. A dual-channel analytical structure integrates financial reporting indicators and risk management information. A gated temporal convolution module models changes in firm-level financial characteristics, while a dynamic graph convolution module represents evolving inter-firm relationships and potential channels of risk transmission. Macroeconomic variables and cross-environment regularization are incorporated to address distribution shifts across economic conditions. SHAP-based analysis is used to interpret the contribution of predictive features and to examine nonlinear patterns and interactions associated with financial vulnerability. The reported empirical comparisons indicate that the proposed framework achieves favorable predictive discrimination, early warning performance, and stability relative to the evaluated baseline models. The study thus presents a unified approach for integrating longitudinal financial data, inter-firm network information, and interpretable machine learning into corporate financial distress assessment.

Index Terms financial distress early warning; temporal–graph fusion; dynamic graph neural network; risk propagation; distribution drift

I. Introduction

Amid growing global economic uncertainty, industrial restructuring, and increasingly interconnected supply chains, advanced manufacturing enterprises face multifaceted financial risks that can develop gradually and spread through business relationships [1]. Compared with firms operating in less capital-intensive sectors, advanced manufacturers often depend on substantial fixed investment, sustained research and development expenditure, specialized technologies, and coordinated supplier networks. These characteristics can increase their exposure to demand shocks, liquidity constraints, and disruptions involving upstream and downstream partners. Financial difficulties at a strategically important firm may affect other enterprises through delayed payments, disrupted orders, or contractual dependencies, potentially amplifying vulnerabilities across a wider industrial network [2], [3]. Consequently, the development of accurate, interpretable, and forward-looking financial distress early warning systems is important for corporate risk management, supply chain resilience, and financial stability [4].

Research on corporate financial distress prediction has progressed from traditional statistical techniques to machine learning and, more recently, deep learning. Classical approaches, including Altman’s Z-score model and logistic

regression-based bankruptcy prediction, assess financial vulnerability using combinations of accounting ratios and related explanatory variables [5], [6]. Their relative transparency makes them useful as benchmarks, but linear specifications and restrictive modeling assumptions can limit their ability to capture nonlinear interactions, heterogeneous firm characteristics, and changes in financial conditions over time. Machine learning methods, such as support vector machines, decision trees, and random forests, can accommodate more complex relationships and may improve classification performance. Nevertheless, their application often relies on static or separately constructed observations, which may inadequately represent the sequence through which financial conditions deteriorate [7].

Temporal deep learning architectures offer a means of modeling these evolving patterns. Recurrent neural networks, long short-term memory networks, temporal convolutional networks, and attention-based architectures can extract information from sequences of financial indicators and identify potentially important changes before an observed distress event. Studies using multiperiod corporate reports likewise demonstrate the relevance of combining information across reporting periods [8]. Yet a model focused exclusively on individual enterprise histories may overlook economically

meaningful relationships among firms. Advanced manufacturing enterprises interact through supply contracts, equity ownership, credit arrangements, and shared industrial exposures. These connections provide potential channels through which operational or financial disruptions can influence otherwise distinct enterprises [9], [10]. Accordingly, temporal prediction and relational risk modeling should be considered jointly rather than as isolated analytical tasks.

Graph neural networks offer a flexible framework for representing inter-firm dependencies by treating enterprises as nodes and their business or financial relationships as weighted edges. Such approaches can aggregate information from connected firms and account for the location of an enterprise within a broader risk network [11]. However, the composition and intensity of inter-firm links can change as supplier contracts, financing arrangements, and market conditions evolve. Models based on a fixed graph may therefore miss changes in network exposure, while purely graph-based representations may not fully capture the trajectory of an individual firm's financial indicators. Two additional challenges complicate practical deployment. First, shifts in macroeconomic conditions can change the statistical distribution of the inputs and weaken performance when models are applied outside their training environment. Second, complex prediction systems may be difficult for managers or regulators to interpret, even when their classification accuracy is favorable. Explainability methods, including SHAP, can help reveal which recorded features contribute to individual predictions, although such feature attributions should not be interpreted as causal effects [12], [13].

To address these challenges, this study develops a temporal-graph fusion framework for financial distress early warning in advanced manufacturing enterprises. The proposed approach first integrates financial reporting indicators and risk management characteristics through a dual-channel analytical structure. It then combines gated temporal convolutions, which capture changes in firm-level financial sequences, with dynamic graph learning, which represents evolving relationships and associated risk exposures among enterprises. Macroeconomic indicators and a cross-environment regularization objective are incorporated to address variation between economic settings, while SHAP-based analysis is used to examine the contribution of model inputs to predictions. The framework is evaluated against conventional classification approaches using discrimination and early warning measures. Its central contribution is the integration of longitudinal financial information, time-varying inter-firm structure, and interpretable prediction within a unified modeling approach intended to support more informed assessment of corporate financial vulnerability.

II. Methodology

A. A Dual-Channel Analysis Framework for Financial Distress Early Warning

Figure 1 illustrates the proposed financial distress early warning framework for advanced manufacturing enterprises. Over-

all, the framework consists of two complementary analytical channels: the upper part is the financial reporting analysis channel, and the lower part is the risk management analysis channel. The former characterizes the current financial status and operating performance of the enterprise, while the latter identifies potential risk exposures and risk transmission mechanisms. Together, they establish the analytical foundation for financial distress early warning model presented in this paper [14].

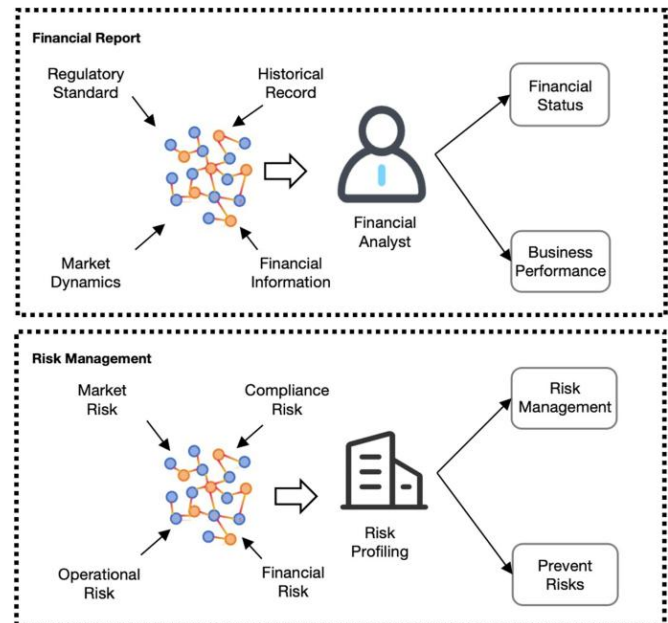


Figure 1: Dual-Channel Framework for Financial Distress Early Warning in Advanced Manufacturing Enterprises

In the financial reporting analysis channel, the model comprehensively utilizes multi-source data, including regulatory standards, historical financial records, market dynamics, and enterprise financial information. After feature extraction and temporal modeling, this data forms a representation of the enterprise's financial status, used to determine whether the enterprise faces problems such as declining solvency, weakened profitability, deteriorating cash flow, or declining operating performance. This channel therefore supports the assessment of the enterprise's financial status and operating performance.

In the complementary risk management channel, the framework considers market, compliance, operational, and financial exposures to construct a broader risk profile for each enterprise. Unlike approaches restricted to accounting indicators, this channel also incorporates exposures associated with supply chain disruptions, research and development expenditure, inventory accumulation, changes in customer demand, and shifts in regulatory requirements. The resulting information complements financial statement analysis and may help identify vulnerabilities that are not yet reflected in reported ratios [15].

Accordingly, the proposed framework treats financial distress early warning as a comprehensive prediction problem

rather than a classification exercise based solely on accounting ratios. It combines internal financial characteristics, broader risk exposures, temporal dynamics, and inter-firm dependencies, while explicitly considering potential changes in data distributions. Figure 1 summarizes this design and establishes the basis for the deep learning architecture presented below.

Let the financial reporting characteristics of firm i at time t be $\mathbf{x}_{i,t}^F$, the risk management characteristics be $\mathbf{x}_{i,t}^R$, and the inter-firm relationship graph be $\mathcal{G}_t = (\mathcal{V}, \mathcal{E}_t)$. The comprehensive risk representation after dual-channel fusion can be defined as:

$$\mathbf{h}_{i,t} = \phi(\mathbf{W}_f \cdot \text{Attn}_T(\mathbf{x}_{i,t-L:t}^F) + \mathbf{W}_r \cdot \text{GNN}(\mathbf{x}_{i,t-L:t}^R, \mathcal{G}_t) + \mathbf{W}_m \cdot \mathbf{m}_t + \mathbf{b}), \quad (1)$$

where $\mathbf{h}_{i,t}$ represents the comprehensive risk representation of firm i at time t ; $\text{Attn}_T(\cdot)$ is used to capture the time dependencies in historical financial reports; $\text{GNN}(\cdot)$ is used to model potential risk transmission relationships between firms; $\mathbf{m}_t$ represents macroeconomic and market dynamic variables; $\mathbf{W}_f$, $\mathbf{W}_r$, and $\mathbf{W}_m$ are learnable parameters; and $\phi(\cdot)$ is a nonlinear activation function.

Furthermore, to improve the model's generalization ability under different economic cycles and market environments, this paper introduces a cross-environment robust optimization objective. Let the set of different economic environments be $\mathcal{E}$, then the model optimization objective can be expressed as:

$$\min_{\Theta} \left\{ \begin{aligned} & \sum_{e \in \mathcal{E}} \frac{1}{N_e} \sum_{i=1}^{N_e} \mathcal{L}(y_{i,t+\Delta}^e, \sigma(\mathbf{w}^\top \mathbf{h}_{i,t}^e)) \\ & + \lambda_1 \sum_{e, e' \in \mathcal{E}} \|\mathbb{E}[\mathbf{h}_{i,t}^e | y] - \mathbb{E}[\mathbf{h}_{i,t}^{e'} | y]\|_2^2 \\ & + \lambda_2 \|\Theta\|_2^2 \end{aligned} \right\} \quad (2)$$

where $y_{i,t+\Delta}^e$ represents whether firm i will experience financial distress over the subsequent Δ periods; $\sigma(\cdot)$ is the Sigmoid function; $\mathcal{L}(\cdot)$ is the binary cross-entropy loss; the first term measures predictive error, the second term is used to constrain the consistency of risk representation of similar firms under different economic environments, thereby alleviating the distribution drift problem, and the third term is the model regularization term.

B. Dynamic Enterprise Graph Modeling

To characterize the complex relationships among enterprises in advanced manufacturing and their evolution over time, this paper introduces a Dynamic Enterprise Graph (DEP) modeling mechanism, as shown in Figure 2. The figure illustrates the evolution of the enterprise relationship network across different time slices, where nodes represent individual enterprises and edges represent relationships between enterprises, such as supply chain dependencies, equity structures, or industry similarities [16].

Unlike a static graph, the proposed representation allows the existence and strength of inter-firm connections to vary across observation periods. Changes in supplier agreements,

ownership arrangements, or shared market exposures can therefore be reflected in a sequence of network snapshots. Formally, enterprise relationships are represented by time-indexed graph structures:

$$\mathcal{G}_t = (\mathcal{V}, \mathcal{E}_t), \quad t = 1, 2, \dots, T, \quad (3)$$

where $\mathcal{V}$ represents the set of enterprise nodes, and $\mathcal{E}_t$ represents the set of edges at time t . This modeling approach can reflect the structural changes exhibited by advanced manufacturing enterprises during different economic cycles, market environments, and supply chain adjustments.

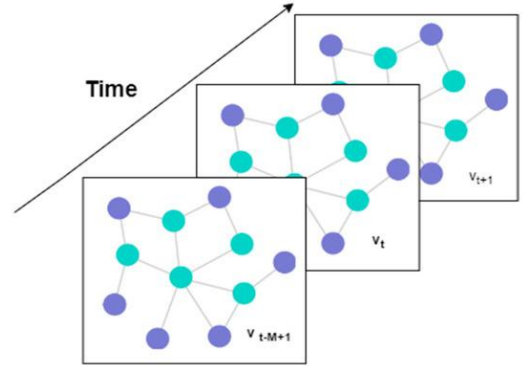


Figure 2: Temporal Evolution of the Inter-Firm Relationship Network

Based on this, this paper further considers the propagation effect of risk in the enterprise network. Because enterprises are linked through supply chain dependencies and financial relationships, difficulties at one firm may spread to upstream and downstream enterprises through the network structure. Therefore, a graph neural network is introduced to update the node representation:

$$\mathbf{z}_{i,t} = \sum_{j \in N_i^t} \alpha_{ij}^t W h_{j,t-1}, \quad (4)$$

where $\mathbf{z}_{i,t}$ represents the graph structure representation of firm i at time t ; N_i^t represents the set of neighboring nodes of firm i at time t ; α_{ij}^t represents the influence weight of node (j) on node (i) (which can be learned through an attention mechanism); W are learnable parameters; and $\sigma(\cdot)$ is a nonlinear activation function.

Furthermore, to simultaneously capture the dynamic evolution and structural dependencies in the time dimension, this paper jointly models the graph structures observed at successive historical time steps to construct a temporal-graph representation:

$$\mathbf{H}_{i,t} = \text{TemporalGNN}(\{\mathcal{G}_{t-M+1}, \dots, \mathcal{G}_t\} \{\mathbf{X}_{t-M+1}, \dots, \mathbf{X}_t\}), \quad (5)$$

where $\mathbf{H}_{i,t}$ represents the firm representation after integrating time and structural information; $\mathbf{X}_t$ represents the set of node features at time t . This model can simultaneously learn:

the temporal evolution of a firm's own financial status; the structural path of risk propagation between firms; and the dynamic impact of changes in network structure.

As illustrated in Figure 2, the network may undergo structural reorganization, with connections appearing, disappearing, or changing in strength between observation periods. Such temporal variation is relevant in advanced manufacturing, where supplier replacement, demand fluctuations, and regulatory adjustments can alter the relationships through which firms are exposed to one another. The use of multiple graph snapshots allows these changes to be represented explicitly.

C. A Deep Financial Distress Early Warning Model Based on Temporal–Graph Fusion

Figure 3 illustrates the core model proposed in this paper. The overall process can be understood as follows: First, multi-source inputs are constructed from historical financial data, market dynamics data, risk management data, and inter-firm relationship data. Then, the evolutionary characteristics of the enterprise's own state and the risk propagation characteristics between enterprises are extracted through a temporal feature extraction module and a graph structure learning module, respectively. Finally, a multilayer fusion module produces a prediction of financial distress events [17].

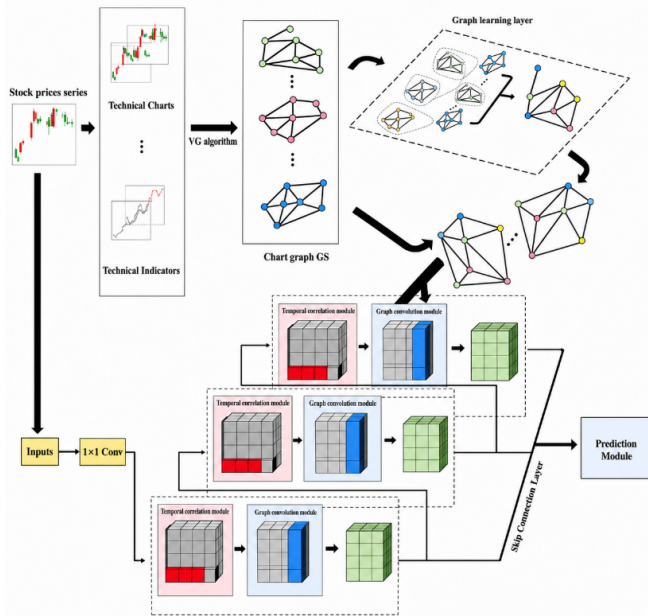


Figure 3: Temporal–Graph Fusion Deep Learning Model

The left-hand component of Figure 3 summarizes the multi-source information associated with each enterprise. Rather than relying exclusively on stock-market observations, the proposed setting emphasizes successive financial reports, operational indicators, macroeconomic variables, and risk management information. Illustrative features include leverage and solvency ratios, profitability, cash flow, research and

development expenditure, inventory turnover, supplier concentration, and market or compliance exposures. Together, these measurements provide a longitudinal description of the enterprise's financial and operating conditions.

The graph learning section at the top of Figure 3 corresponds to the dynamic enterprise relationship graph modeling module in this paper. Enterprises are considered nodes in the graph, and supply chain relationships, industry similarities, equity relationships, or financial similarities between enterprises are considered edges. Through graph generation algorithms, enterprise relationship graphs can be constructed within different time windows, and the graph learning layer can be used to capture potential risk propagation paths between enterprises. This section answers the question: Will a company's financial risk be affected by its related companies?

The lower component of Figure 3 focuses on temporal representation learning. Stacked convolutional operations identify changes within successive reporting periods, including short-term fluctuations and sustained trends. These patterns may include persistently negative operating cash flow, increasing leverage, slower inventory turnover, and declining revenue growth. Their temporal ordering can be important, because a sequence of deteriorating indicators may convey different information from the same indicators observed at a single point in time.

The prediction component on the right side of Figure 3 integrates temporal, structural, and risk profile representations to estimate the probability of a financial distress event over a specified future horizon, denoted by Δ . Attention weights may indicate how the model distributes information across graph connections, whereas SHAP attribution can provide a complementary assessment of feature contributions. Neither type of explanation alone establishes a causal mechanism.

1) Multi-Source Input Representation

Let the multi-source characteristics of firm i at time t be:

$$\mathbf{x}_{i,t} = [\mathbf{x}_{i,t}^F; \mathbf{x}_{i,t}^R; \mathbf{x}_{i,t}^M; \mathbf{x}_{i,t}^S], \quad (6)$$

where $\mathbf{x}_{i,t}^F$ represents financial reporting characteristics, $\mathbf{x}_{i,t}^R$ represents risk management characteristics, $\mathbf{x}_{i,t}^M$ represents macroeconomic market characteristics, and $\mathbf{x}_{i,t}^S$ represents supply chain or enterprise relationship characteristics. For a historical observation window of length L , the time-series input of firm i can be represented as:

$$\mathbf{X}_{i,t-L:t} [\mathbf{x}_{i,t-L}, \mathbf{x}_{i,t-L+1}, \dots, \mathbf{x}_{i,t}] \in \mathbb{R}^{L \times d}. \quad (7)$$

2) Temporal Feature Extraction Module

To capture the temporal evolution of a company's financial status, this paper employs a time-series encoder to extract dynamic change patterns within a historical window:

$$\begin{aligned}\mathbf{T}_{i,t} &= \text{TCN}(\mathbf{X}_{i,t-L:t}) \\ &= \phi \left(\sum_{k=0}^{K-1} \mathbf{W}_k^T \mathbf{x}_{i,t-k} + \mathbf{b}^T \right),\end{aligned}\quad (8)$$

where $\mathbf{T}_{i,t}$ represents the temporal feature representation of enterprise i ; $\text{TCN}(\cdot)$ represents the temporal convolution module; (K) is the kernel size; $\mathbf{W}_k^T$ are the temporal convolution parameters; and $\phi(\cdot)$ is the nonlinear activation function.

This module is designed to capture the changing trends of an enterprise's own financial status, such as continuously deteriorating financial indicators, fluctuating operating cash flow, declining revenue growth, and inventory backlog.

3) Dynamic Graph Structure Learning Module

In advanced manufacturing scenarios, corporate financial vulnerabilities do not necessarily arise in isolation but can propagate through supply chains, industry clusters, and capital relationships. Therefore, this paper constructs a dynamic corporate relationship graph within each time window:

$$\mathcal{G}_t = (\mathcal{V}, \mathcal{E}_t, \mathbf{A}_t), \quad (9)$$

where $\mathcal{V}$ represents the set of enterprise nodes, $\mathcal{E}_t$ represents the set of enterprise relationship edges at time t , and $\mathbf{A}_t$ represents the adjacency matrix of the dynamic graph.

The edge weights between enterprise i and enterprise (j) can be determined by financial similarity, supply chain connections, industry ties, and equity relationships:

$$\begin{aligned}a_{ij}^t &= \eta_1 \cdot \text{sim}(\mathbf{x}_{i,t}^F, \mathbf{x}_{j,t}^F) + \eta_2 \cdot r_{ij}^{\text{supply}} \\ &\quad + \eta_3 \cdot r_{ij}^{\text{industry}} + \eta_4 \cdot r_{ij}^{\text{equity}},\end{aligned}\quad (10)$$

where a_{ij}^t represents the strength of the association between firm i and firm j at time t ; $\text{sim}(\cdot)$ represents the similarity of financial characteristics; r_{ij}^{supply} , r_{ij}^{industry} , and r_{ij}^{equity} represent supply chain relationship, industry relationship, and equity relationship, respectively; $\eta_1, \eta_2, \eta_3, \eta_4$ are weight coefficients.

Subsequently, the model learns the relative influence of connected firms on the target firm's learned representation through a graph attention mechanism:

$$\alpha_{ij}^t = \frac{\exp(\text{LeakyReLU}(\mathbf{a}^\top [\mathbf{W}^G \mathbf{T}_{i,t} \mathbf{W}^G \mathbf{T}_{j,t} a_{ij}^t]))}{\sum_{k \in \mathcal{N}_i^t} \exp(\text{LeakyReLU}(\mathbf{a}^\top [\mathbf{W}^G \mathbf{T}_{i,t} \mathbf{W}^G \mathbf{T}_{k,t} a_{ik}^t]))}, \quad (11)$$

where α_{ij}^t represents the risk propagation weight of firm j to firm i ; $\mathcal{N}_i^t$ represents the set of neighboring firms of firm i at time t ; (Vert) represents vector concatenation; $\mathbf{W}^G$ and $\mathbf{a}$ are learnable parameters.

Based on this attention weight, the graph structure risk of firm i is represented as:

$$\mathbf{G}_{i,t} = \sigma \left(\sum_{j \in \mathcal{N}_i^t} \alpha_{ij}^t \mathbf{W}^G \mathbf{T}_{j,t} \right). \quad (12)$$

This formula corresponds to the graph learning layer in the diagram, and is intended to capture relational features relevant to possible risk transmission from the enterprise relationship network.

4) Temporal–Graph Fusion Prediction Module

To combine the firm's financial trajectories and information about relationships and potential risk transmission among firms, this paper designs a fusion representation:

$$\mathbf{z}_{i,t} = \text{Fusion}(\mathbf{T}_{i,t}, \mathbf{G}_{i,t}, \mathbf{x}_{i,t}^R, \mathbf{x}_{i,t}^M). \quad (13)$$

More explicitly, the fusion operation can be written as follows:

$$\begin{aligned}\mathbf{z}_{i,t} &= \phi(\mathbf{W}_T \mathbf{T}_{i,t} + \mathbf{W}_G \mathbf{G}_{i,t} \\ &\quad + \mathbf{W}_R \mathbf{x}_{i,t}^R + \mathbf{W}_M \mathbf{x}_{i,t}^M + \mathbf{b}_z),\end{aligned}\quad (14)$$

where $\mathbf{z}_{i,t}$ represents the integrated risk representation of the enterprise after fusion; $\mathbf{W}_T$, $\mathbf{W}_G$, $\mathbf{W}_R$, and $\mathbf{W}_M$ are mapping matrices of time-series characteristics, graph structure characteristics, risk management characteristics, and macro market characteristics, respectively.

Finally, the probability that enterprise i will experience financial distress over the subsequent Δ periods is defined as:

$$\begin{aligned}\hat{y}_{i,t+\Delta} &= P(y_{i,t+\Delta} = 1 \mid \mathbf{X}_{i,t-L:t}, \mathcal{G}_t) \\ &= \sigma(\mathbf{w}_o^\top \mathbf{z}_{i,t} + b_o),\end{aligned}\quad (15)$$

where $\hat{y}_{i,t+\Delta}$ represents the predicted probability of financial distress; $y_{i,t+\Delta} = 1$ indicates that the company will experience financial distress in the subsequent Δ periods; and $\sigma(\cdot)$ is the Sigmoid function.

5) Overall Optimization Objective

To simultaneously ensure prediction accuracy, structural robustness, and cross-environment generalization ability, the overall loss function of the model in this paper is defined as:

$$\begin{aligned}\mathcal{J} &= -\frac{1}{N} \sum_{i=1}^N [y_{i,t+\Delta} \log \hat{y}_{i,t+\Delta} \\ &\quad + (1 - y_{i,t+\Delta}) \log (1 - \hat{y}_{i,t+\Delta})] \\ &\quad \underbrace{\hspace{10em}}_{\text{Financial distress prediction loss}} \\ &\quad + \lambda_1 \underbrace{\sum_{e, e' \in \mathcal{E}} \|\mathbb{E}[\mathbf{z}_{i,t}^e \mid y] - \mathbb{E}[\mathbf{z}_{i,t}^{e'} \mid y]\|_2^2}_{\text{Distribution shift regularization}} \\ &\quad + \lambda_2 \underbrace{\sum_{t=2}^T \|\mathbf{A}_t - \mathbf{A}_{t-1}\|_F^2 + \lambda_3 \|\Theta\|_2^2}_{\text{Dynamic graph smoothness}},\end{aligned}\quad (16)$$

The loss in Eq. (16) contains four components. The first measures binary classification error. The second penalizes differences in learned representations across economic environments, with the aim of mitigating distribution shift. The third discourages abrupt changes in graph adjacency matrices

between consecutive time steps, and the fourth provides conventional parameter regularization. The relative importance of these objectives is controlled by their associated regularization coefficients.

D. Gated Temporal Graph Convolution Block

The core computational component is the gated temporal graph convolution (GTGC) block illustrated in Figure 4. The block consists of an input projection, a gated temporal convolution, a graph convolutional network (GCN), and residual and skip connections. These components combine financial-sequence information with the relationships among enterprises and permit the resulting representations to be stacked in successive layers. Related spatiotemporal graph architectures provide methodological precedents for integrating convolutional temporal processing with graph-based aggregation [18], [19].

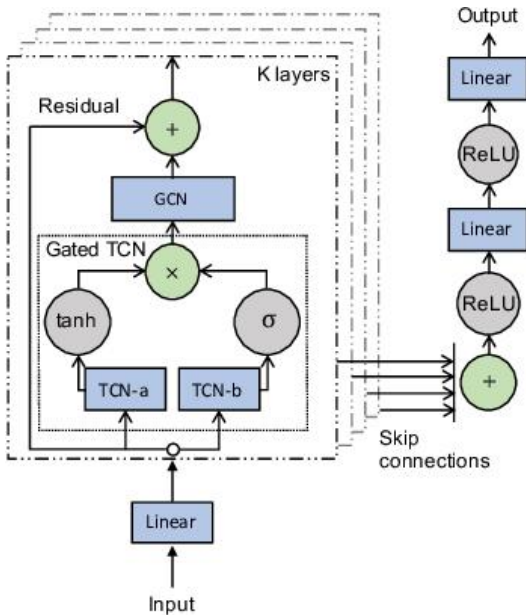


Figure 4: Gated Temporal–Graph Convolution Block

Let the input of the (k)th layer be $\mathbf{H}^{(k)} \in \mathbb{R}^{N \times L \times d}$, where (N) is the number of firms, (L) is the time window length, and (d) is the feature dimension. First, a linear mapping is used to unify the multi-source inputs into the latent space:

$$\mathbf{H}^{(k)} = \text{Linear}(\mathbf{H}^{(k)}). \quad (17)$$

The system then enters the gated temporal convolution and graph convolution modules, forming the main path within each layer.

To extract key change patterns from historical financial sequences, a dual-branch temporal convolutional network (TCN) is used to construct a gating mechanism: one branch generates candidate features, and the other branch generates gating weights. Information filtering is achieved through element-wise multiplication.

$$\mathbf{Q}^{(k)} = \tanh(\text{TCN}_a^{(k)}(\mathbf{H}^{(k)})) \odot \sigma(\text{TCN}_b^{(k)}(\mathbf{H}^{(k)})), \quad (18)$$

where $\text{TCN}_a^{(k)}(\cdot)$ and $\text{TCN}_b^{(k)}(\cdot)$ are two parallel temporal convolution branches, $\sigma(\cdot)$ is the Sigmoid function, and $\odot$ represents element-wise multiplication. This design can adaptively emphasize critical time slices (such as deteriorating cash flow or sudden increases in leverage) while suppressing noise fluctuations.

After obtaining the gated temporal features $\mathbf{Q}^{(k)}$, an enterprise relationship graph $\mathcal{G}_t$ is introduced to aggregate neighbor information, characterizing the propagation effect of risk in the supply chain or industry network. A normalized graph convolution form is used:

$$\mathbf{S}^{(k)} = \phi(\mathbf{D}_t^{-\frac{1}{2}} \mathbf{A}_t \mathbf{D}_t^{-\frac{1}{2}} \mathbf{Q}^{(k)} \mathbf{W}_g^{(k)}), \quad (19)$$

where $\mathbf{A}_t = \mathbf{A}_t + \mathbf{I}$ is the adjacency matrix with added self-loops, $\mathbf{D}_t$ is the correspondence matrix, $\mathbf{W}_g^{(k)}$ is the graph convolution parameter of the (k)th layer, and $\phi(\cdot)$ is the nonlinear activation function. This step explicitly incorporates inter-firm dependencies into the representation learning process.

To alleviate the vanishing-gradient problem in deep network training and preserve information representation at different levels, the module adopts a joint design of residual connections and cross-layer skip connections. Intra-layer updates are as follows:

$$\mathbf{H}^{(k+1)} = \mathbf{S}^{(k)} + \mathbf{H}^{(k)}. \quad (20)$$

Simultaneously, the outputs from successive layers are combined before prediction via skip connections:

$$\mathbf{Z}_t = \sum_k 1^K \mathbf{H}^{(k)} \mathbf{W}_s^{(k)}, \quad (21)$$

where $\mathbf{W}_s^{(k)}$ represents the cross-layer fusion weights. This design enables the model to simultaneously utilize shallow local fluctuation information and deep structural propagation information.

The fusion representation $\mathbf{Z}_t$ is mapped nonlinearly to obtain the probability of the firm experiencing financial distress over the subsequent Δ periods:

$$\hat{y}_{t+\Delta} = \sigma(\mathbf{W}_o^{(2)} \text{ReLU}(\mathbf{W}_o^{(1)} \text{ReLU}(\mathbf{Z}_t) + \mathbf{b}_o^{(1)}) + \mathbf{b}_o^{(2)}). \quad (22)$$

This output corresponds to the prediction module on the right side of Figure 4.

In summary, the GTGC block in Figure 4 is designed to integrate two complementary sources of information. Gated temporal convolutions capture changes in the firm-level financial sequence, while graph convolution aggregates relevant information from connected enterprises. Residual and cross-layer skip connections retain representations from different depths and are intended to support optimization of the stacked architecture. Their combined use enables the model to represent both longitudinal financial conditions and network-related exposures.

III. Experiments and Results

A. Experimental Setup

To systematically verify the effectiveness and superiority of the proposed temporal–graph fusion financial distress early warning model in advanced manufacturing enterprise scenarios, this paper evaluates the framework using multi-source information and examines performance from multiple dimensions, including overall performance, early warning capability, the role of macroeconomic variables, risk propagation mechanisms, and model interpretability [15].

The experimental data encompass enterprise financial reporting information, risk management indicators, macroeconomic variables, and enterprise relationship network data. Financial data includes debt-to-equity ratio, cash flow indicators, profitability, and operational efficiency; risk management data covers market risk, compliance risk, and operational risk; macroeconomic variables include credit expansion, debt-to-service ratio (DSR), and GDP growth; and enterprise relationship data is constructed using a dynamic enterprise relationship graph based on supply chain, industry linkages, and financial similarities. To reduce temporal information leakage, the study uses chronological data partitioning to construct the training, validation, and test sets, thereby providing a more realistic evaluation of performance on subsequent observations. Model performance is evaluated using AUC, F1-score, Recall, and KS statistics, with Recall serving as a key indicator to measure the model’s ability to identify high-risk enterprises [4].

B. Experimental Results

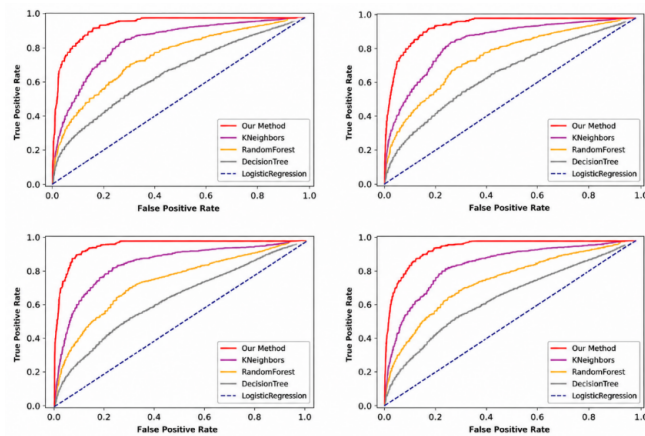


Figure 5: Comparison of ROC Curves for Financial Distress Prediction Models

Figure 5 compares receiver operating characteristic (ROC) curves of different models in financial distress early warning task. Overall, all models outperform the random classification baseline, but significant performance differences exist among the methods. The traditional linear model, Logistic Regression, performs the weakest, indicating its inability to characterize the nonlinear features of financial risk. While the

Decision Tree model can capture local patterns, its overall stability is poor. Random Forest improves performance through ensemble learning but still struggles to model complex temporal dependencies. k-nearest neighbors performs well in local structures but lacks global generalization ability. In contrast, the proposed method is reported to perform favorably in the displayed comparisons, with its ROC curve consistently closest to the top left corner, demonstrating that the model achieves a high true positive rate with a low false positive rate.

Figure 6 illustrates the distribution of risk variables and the correlations between variables in financial distress sample at different time steps. Panel A represents the distribution of distressed companies’ asset size over time, panel B represents the distribution of distressed companies’ credit association strength over time, and panel C represents the relationship between credit association strength and distressed companies’ asset size. Overall, the sample points in rows A and B are mainly concentrated in earlier time steps, indicating that signals associated with corporate financial distress appear to cluster within a specific historical window, rather than being evenly distributed throughout the observation period. Simultaneously, both asset size and credit association strength exhibit considerable dispersion in the early stages, indicating significant heterogeneity among different companies during the crisis formation process. Panel C further shows a positive association between credit association strength and distressed companies’ asset size; that is, companies with larger asset sizes often have higher credit association exposure. This implies that financial distress of advanced manufacturing companies not only stems from the deterioration of their own financial condition but may also be closely related to the strength of their connections in the industrial chain, financing chain, and corporate relationship network.

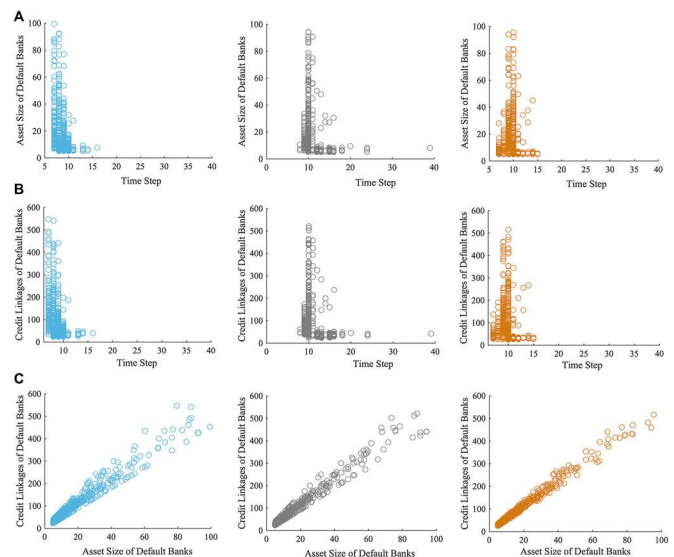


Figure 6: Distribution of Risk Variables Across Time and Firm Asset Size

Figure 7 illustrates the early warning performance of dif-

ferent macroeconomic financial variables across multiple time windows preceding financial distress. The horizontal axis indicates the lead time before the distress event, the vertical axis represents the model’s predictive performance or early warning accuracy, and the shaded region represents the 95% confidence interval. Overall, the credit-to-GDP gap, debt-service ratio (DSR), and noncore liabilities maintained high and relatively stable early warning capabilities over a considerable period before the crisis. In particular, DSR showed a continuous upward trend as the crisis approached, indicating that changes in corporate debt repayment pressure and financing structure could reflect potential financial vulnerabilities early on. In contrast, Credit growth, Property price growth, and GDP growth showed a significant decline in the period leading up to the crisis, suggesting that relying solely on growth-related macroeconomic variables is insufficient to maintain stable identification capabilities before a crisis, and their predictive contributions are easily affected by economic cycle fluctuations and changes in the market environment. This result is consistent with financial distress early warning framework proposed in this paper: the formation of crises in advanced manufacturing enterprises is influenced not only by internal financial indicators but also by the combined effects of macroeconomic financial conditions, debt pressure, credit expansion, and economic cycle changes.

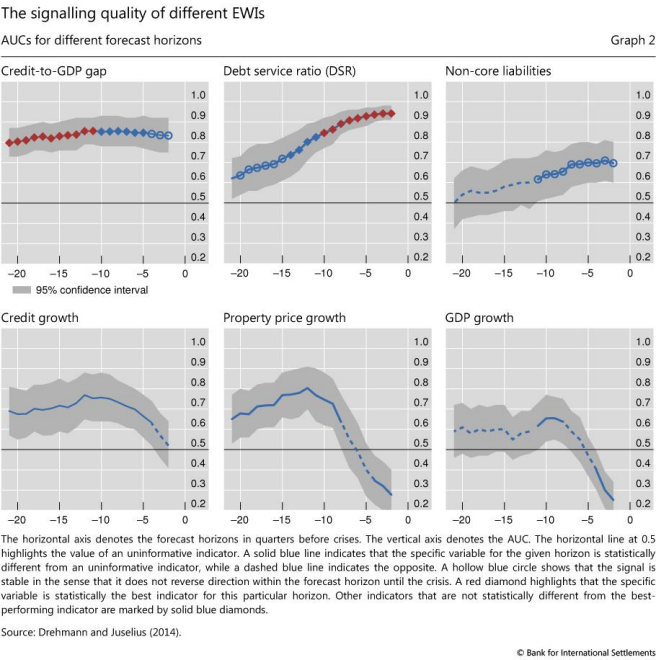


Figure 7: Trends in Early Warning Performance

Figure 8 illustrates the dynamic evolution of the systemic risk contagion network among enterprises across different years. Nodes represent individual enterprises, edges represent risk associations or potential risk transmission relationships between enterprises, and node size reflects the degree of influence or risk exposure of an enterprise within the risk

network. Overall, from 2016 to 2019, the topology of the risk contagion network was not static but exhibited significant time-varying and structural reorganization characteristics. Some core nodes remained at the network center or in highly connected positions for extended periods, indicating their stronger influence in the propagation of systemic risks. Simultaneously, differences in node connection density and network aggregation structure across different years suggest that risk associations among enterprises dynamically adjust with changes in market environment, supply chain relationships, and financial status. This result is highly consistent with the dynamic enterprise relationship graph modeling concept proposed in this paper, namely, that financial distress events in advanced manufacturing enterprises do not occur in isolation but may spread within the enterprise network through supply chain dependence, industry linkages, financing relationships, and operational connections.

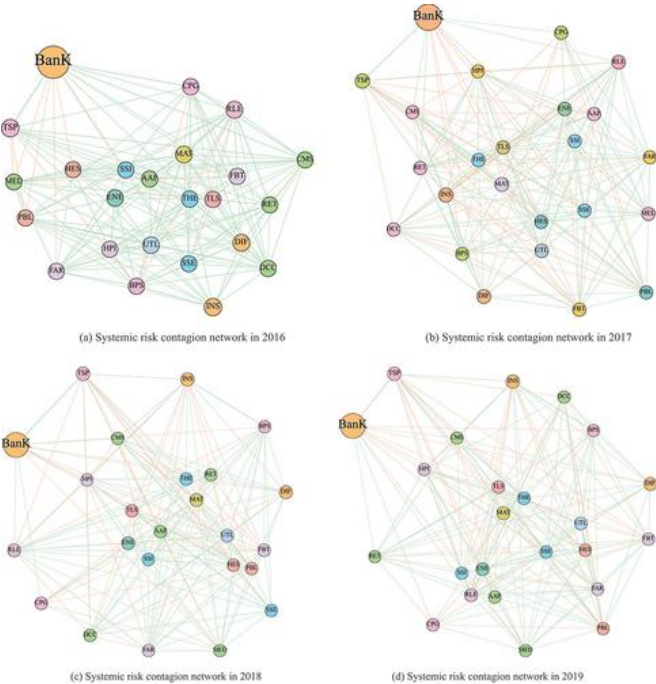


Figure 8: Inter-Firm Systemic Risk Contagion Network

Figure 9 illustrates the dynamic changes in the intensity of systemic risk contagion among enterprises from 2016 to 2019. The horizontal and vertical axes represent enterprise nodes, and the color intensity indicates the strength of risk association between pairs of enterprises. Overall, the heatmaps from different years show significant differences, indicating that the risk contagion relationship among enterprises is not stable but dynamically adjusts with changes in the market environment, industrial chain structure, financing conditions, and enterprise financial status. Blue areas represent strong positive risk associations, suggesting a high degree of shared risk exposure or risk resonance effect among related enterprises; red areas reflect negative associations or risk differentiation relationships, indicating that some enterprises have opposite risk trends

during specific periods. The changes from 2016 to 2019 show that the risk association matrix formed a relatively obvious block structure in some areas, indicating that enterprise risk propagation has certain group aggregation and industry linkage characteristics. This result is consistent with the dynamic enterprise relationship graph modeling and graph convolution risk propagation mechanism proposed in this paper, indicating that financial distress of advanced manufacturing enterprises is not only affected by their own financial situation but also by the combined effects of changes in the risk status and network structure of related enterprises.

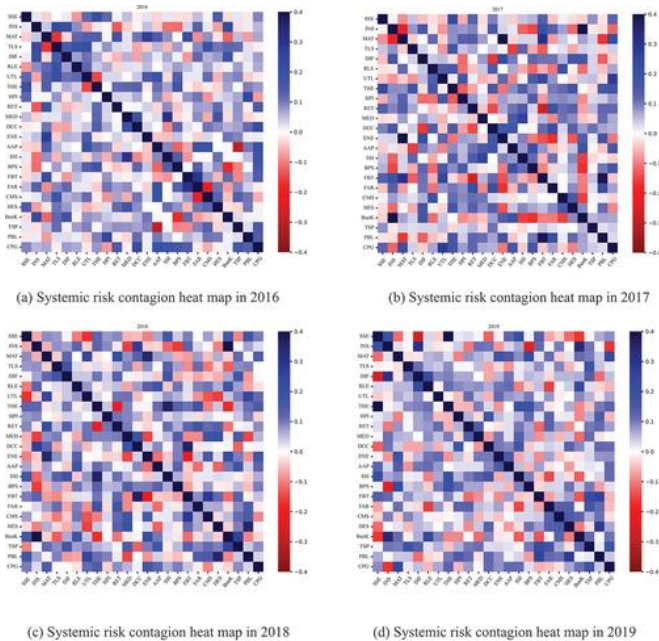


Figure 9: Heatmap of Risk Transmission Intensity

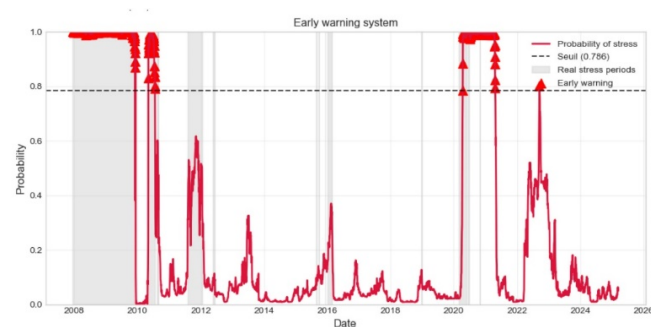


Figure 10: Dynamic Changes in Predicted Financial Distress Probability

Figure 10 illustrates the early warning results of financial distress events using the model presented in this paper over long time series. The red curve represents the probability of financial stress for enterprises or market entities, the dashed black line marks the warning threshold, the gray shading indicates the observed distress period, and red triangular markers denote the early warning signal triggered by the

model. Overall, the model-predicted stress probability rises significantly before several actual crisis intervals and triggers timely warnings after exceeding the threshold, demonstrating that the method presented in this paper not only identifies financial distress events that have already occurred but also has the ability to capture the risk accumulation process in advance. Especially before a crisis occurs, the stress probability curve typically shows a trend of rapid increase from low levels to high levels, indicating that corporate financial risk does not form instantaneously but accumulates gradually with debt repayment pressure, operational fluctuations, market shocks, and the spread of related risks.

Figure 11 illustrates the nonlinear impact of key financial variables on the output of financial distress early warning model presented in this paper. The horizontal axis represents the variable values, and the vertical axis represents the corresponding SHAP values. A higher SHAP value indicates a stronger positive contribution of the variable to the predicted probability of corporate financial distress. Overall, CAR, TAR, RRT, and EM all exhibit significant nonlinearity and threshold effects on the model's prediction results, indicating that corporate financial distress is not necessarily determined by the linear change of a single variable, but rather by the joint triggering of risk by multiple financial indicators within a specific range. Specifically, CAR corresponds to a more positive SHAP attribution at the lower end of its observed range, indicating that the corresponding levels of the measured capital-related indicator are more likely to increase the probability of financial distress; TAR's contribution to risk prediction significantly increases when approaching low values or abnormal ranges, indicating that a decline in asset returns or asset efficiency may be an important signal of deteriorating business operations; RRT shows an abrupt change in SHAP attribution when approaching the critical range, reflecting the existence of risk thresholds in debt repayment, turnover, or profitability-related indicators; EM's overall contribution is relatively small, but it still affects the model's judgment under extreme values.

Figure 12 presents SHAP dependence and interaction effects of key variables in financial distress early warning model presented in this paper. The left panel illustrates the association between an individual feature and its SHAP attribution, while the right panel uses color to indicate another variable that may be associated with the observed attribution pattern. The dependence plots suggest that the relationship between selected financial variables and their SHAP attributions may be nonlinear, with changes in magnitude over certain observed ranges. These visualizations are consistent with the possibility that model predictions depend jointly on solvency, asset efficiency, profitability, operating scale, cash flow pressure, and external conditions. They do not, on their own, establish causal interactions. Specifically, when the core financial variable exceeds a certain critical range, its contribution to the probability of crisis rapidly increases, indicating that the model can identify the key threshold for enterprises to transition from a normal operating state to a high-risk state.

Furthermore, the interaction effects reflected by the color distribution further illustrate that the intensity of the impact of the same financial variable on risk prediction varies under different enterprise sizes, asset structures, or operating cycles.

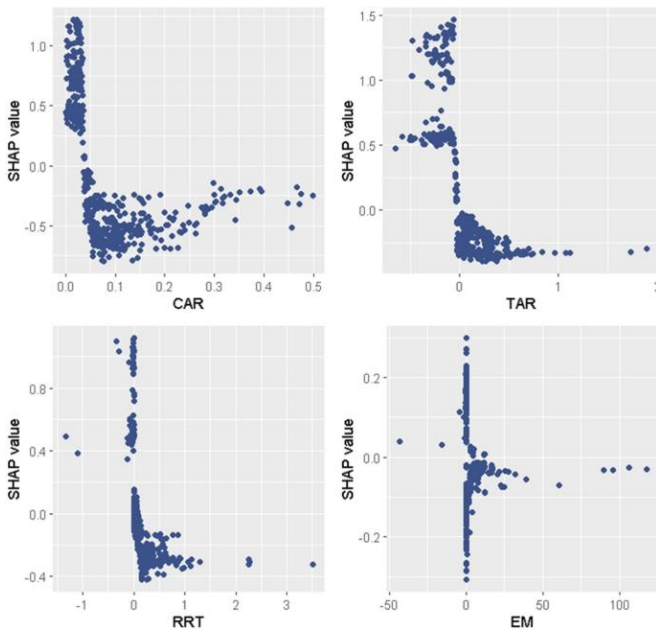


Figure 11: SHAP Dependence Plots for Key Financial Variables

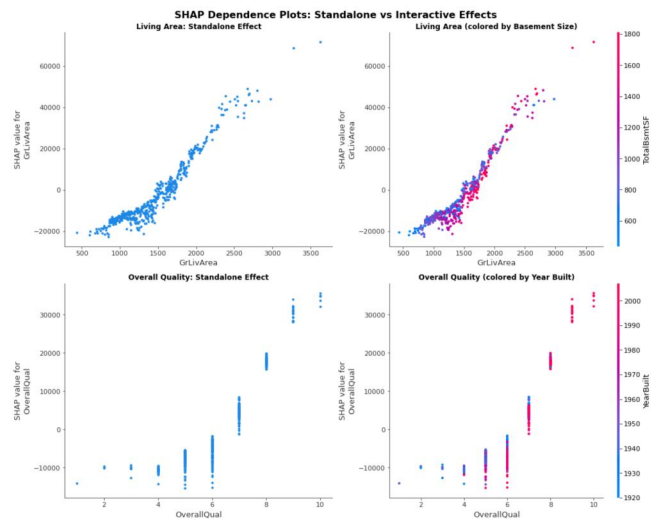


Figure 12: SHAP Dependence and Interaction Effects for Key Financial Variables

IV. Conclusion

This study develops a temporal–graph fusion deep learning framework for early warning of financial distress among advanced manufacturing enterprises. It combines a dual-channel analytical structure for financial reporting and broader risk management information with gated temporal convolutions

and dynamic graph learning. The resulting architecture is designed to integrate changes in an enterprise’s financial indicators with its evolving position in an inter-firm relationship network. A cross-environment regularization component addresses potential shifts in macroeconomic conditions, while SHAP-based attribution supports interpretation of the features contributing to model predictions.

The reported experiments indicate favorable discrimination and early warning performance relative to the evaluated baseline methods. The network analysis also suggests that inter-firm relationships may contain predictive information that is not captured by the financial history of the focal firm alone. These results motivate further evaluation of temporal and structural information fusion in financial distress prediction. However, broader claims about generalization, economic benefits, or causal risk transmission require additional validation using independently documented datasets, clearly defined distress outcomes, and robust out-of-period testing. Future studies should examine model calibration, performance under changing market conditions, and the reliability of explanations across distinct manufacturing industries.

Conflicts of Interest

The author declares no conflicts of interest.

Data Availability

The data are available from the author.

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